Computational Challenges of Global Astrometry

B. Bucciarelli
INAF-OATo

Outline

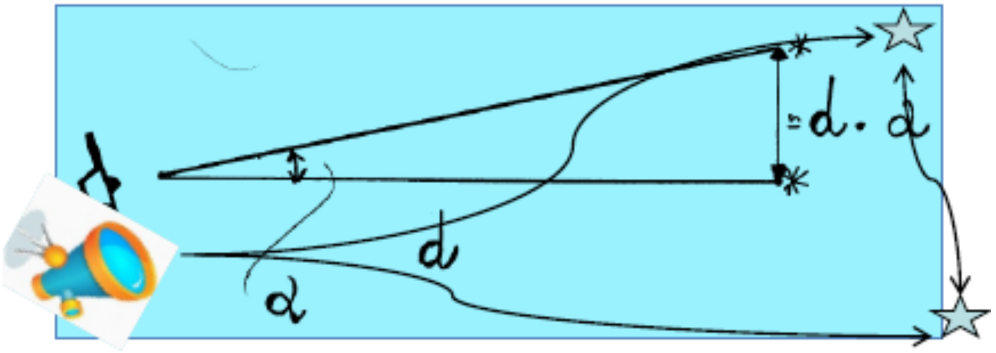
- Objectives of Astrometry
- Global Space Astrometry (HIPPARCOS & Gaia)
- The Sphere Solution in the Gaia Mission
- Analysis of Astrometric Signatures

Objectives of Astrometry

- Study of motion of celestial objects through space with direct impact on several branches of astronomy and fundamental physics
  - Solar System (accurate modeling of planetary motions → Solar system exploration)
  - Milky Way Galaxy (kinematics, dynamics, Galaxy models constraints)
  - Cosmology (extragalactic proper motion signatures → primordial GW)
  - Gravitational Theories (tests of γ and β PPN parameters, tests of Equivalence Principle)

- Basic Observation
  - Measurement of angles between incoming light from celestial sources

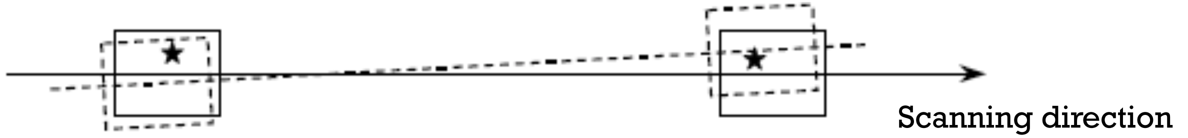
- Methods
  - Small-Field-of-View (< 1°)
  - Large-Field-of-View (few degrees)
  - Global Astrometry (all sky, only from space)



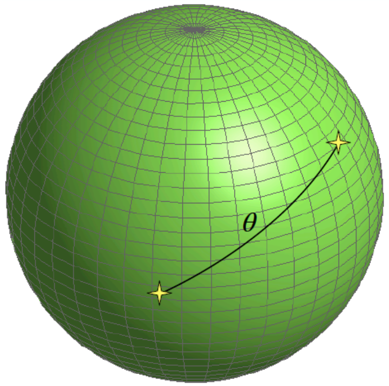
Global Space Astrometry

Principles

- Continously scanning telescope → positional information given by **timing data**
- Simultaneous observation of stars separated by large angles → **absolute parallaxes**
- The observation time is translated into a position of the star relative to the satellite axes
  - Along scan (AL) more important → tolerance on across (AC) measurement ~100 times more relaxed



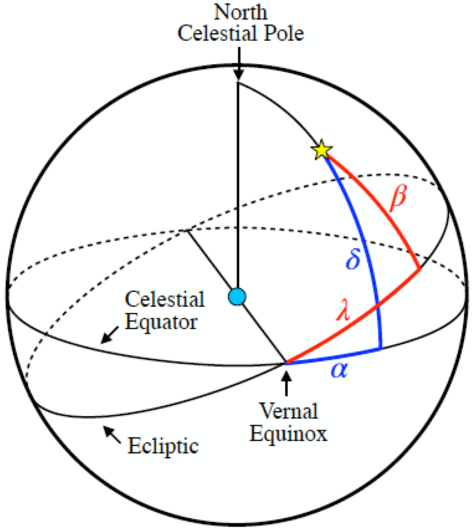
- A very large number of observation times are used to reconstruct an accurate celestial network, which involves also the modeling of satellite instrument and attitude
- The goal is to realize an astrometric catalog such that the angular separation ϑ between any two objects can be computed with an uncertainy which is essentially independent on ϑ
- Such a catalog defines an *undistorted reference frame*
  - It can however be rotating → can be made quasi-inertial using extragalactic objects (quasars)



Global Space Astrometry

The 5-parameter model

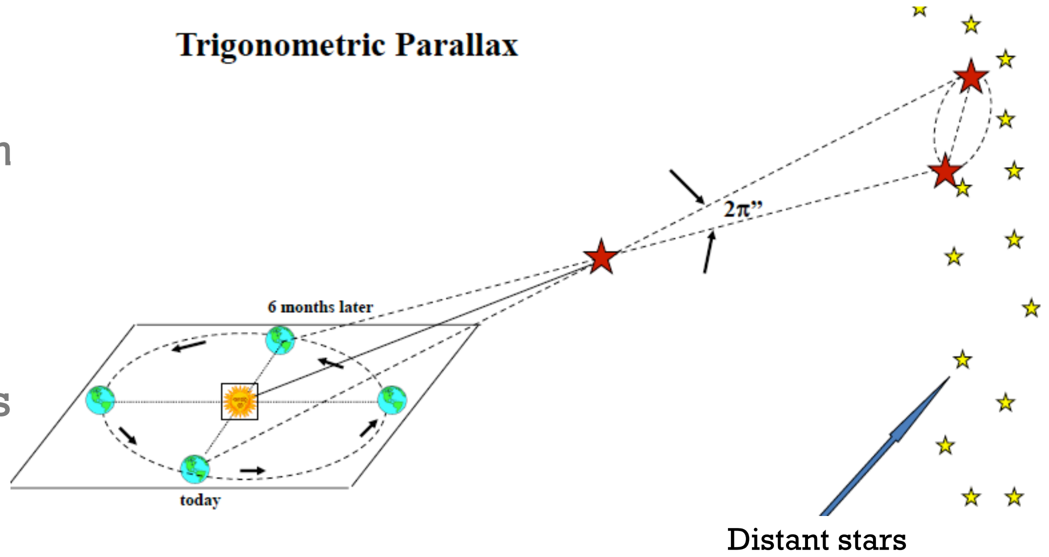
- α_0, δ_0 **direction** on the celestial sphere at the catalog reference epoch
- μ_α, μ_δ **proper motions** $(\frac{d\alpha}{dt}, \frac{d\delta}{dt})$ along α and δ
- ϖ **parallax**



The stellar parallax

- Yearly apparent displacement due to the Earth's motion around the Sun
  - the star describes an apparent ellipse in the sky
  - the semi-major axis of this ellipse is the

    trigonometric parallax ϖ
  - Funciontally related to the star's distance from the Sun



$$D = {}^{1}/_{\varpi}$$

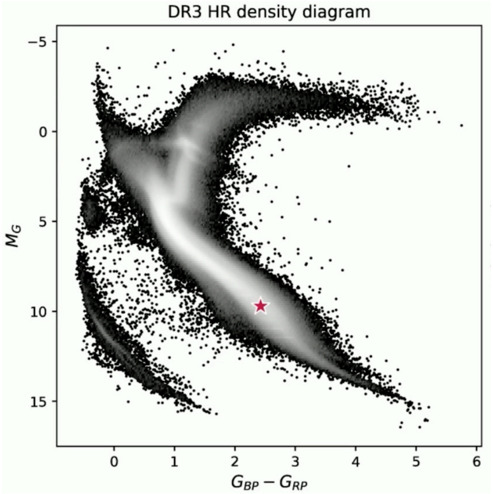
where ϖ is expressed in arcseconds, and D in parsecs
(1 parsec = 206265 Astronomical Units)

Global Space Astrometry

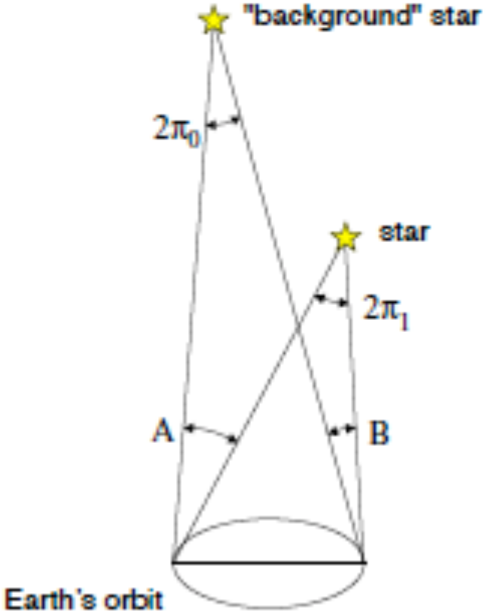
Relative versus Absolute Parallax

- Relative parallaxes (from ground measurements) are referred to background reference stars
- Absolute parallaxes can be determined from simultaneus measurements of stars separated by large angles → only from space
- Absolute (trigonometric) parallaxes are a proxy for **stellar distances**

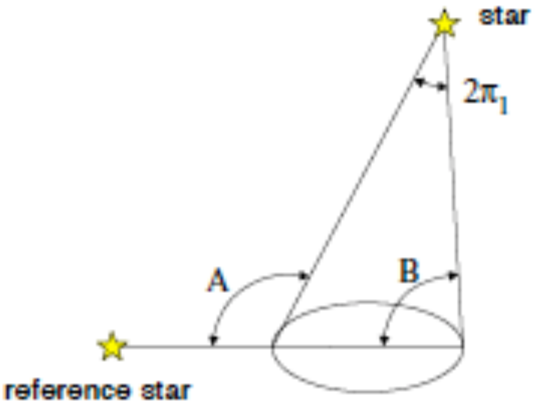
  - Direct distance calibrators



Hertzsprung-Russel diagram

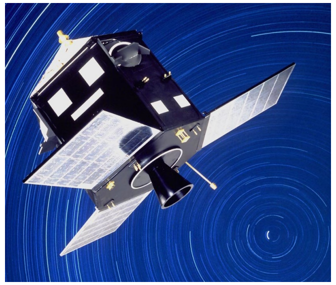


relative parallax

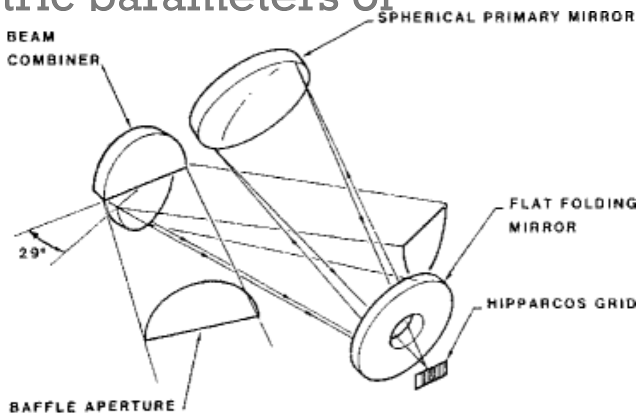


absolute parallax

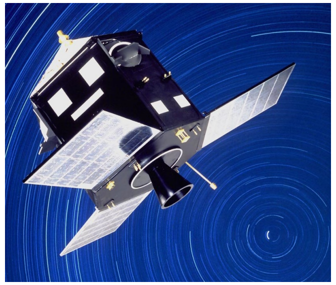
ESA HIPPARCOS Mission



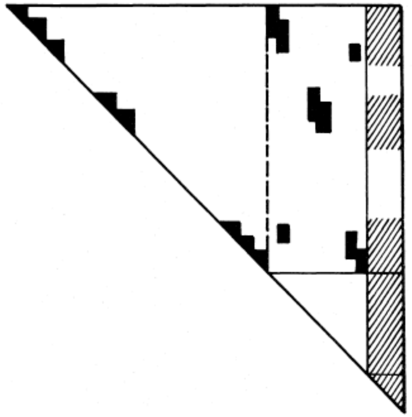
- First space satellite dedicated to astrometry (Launch Aug 1989-End operations Aug 1993)
  - Scanning satellite, simultaneously observing two 0.9°x0.9° FOVs 58° apart by means of a double-mirror beam combiner
  - Incoming star light modulated by grid slits perpendicular to scan direction and sampled by an image-dissector tube photomultipler
- Goal: 5 astrometric parameters of ~120,000 primary stars up to mag~12 with precision of ~2-4 milli-arcseconds
- Data reduction (FAST & NDAC Consortia) was a formidable problem at the time:
  - ~370,000 stellar unknowns and ~2M calibration parameters (attitude+istrumental)
  - Direct solution by elimination of stellar unknowns would have required $\sim n^3/3 = 10^{16}$ flops, plus the administration of $\sim n^2/2 = 6\ 10^{10}$ double-precision reals (~ 500 GB)
- The so-called 3-step method (devised by L. Lindegren) allowed to meet the available computer resources
  - The core step was the Sphere Solution, which determined the 5 astrometric parameters of all the primary stars starting from their abscissae estimated from the great-circle reduction step.
  - The sphere system dimensions were ~400,000 unknowns and ~2M observation equations



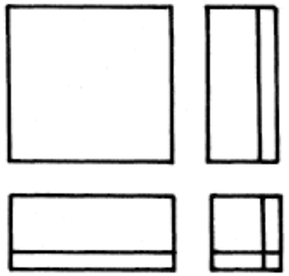
ESA HIPPARCOS Mission



- Numerical Approach to Sphere Solution in FAST (Galligani et al. 1989)
  - Two basic procedures were developed:
    - 2x2 block Cholesky factorization by considering the splitting of the normal matrix
    - computationally more expensive



Normal matrix (NxN)



2x2 splitting

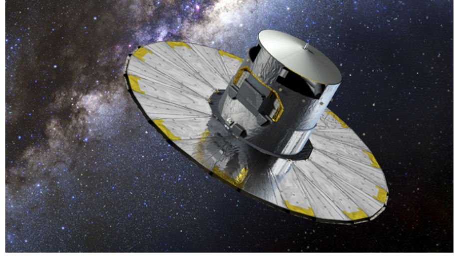
Condition matrix (MxN)

N=number of unknowns

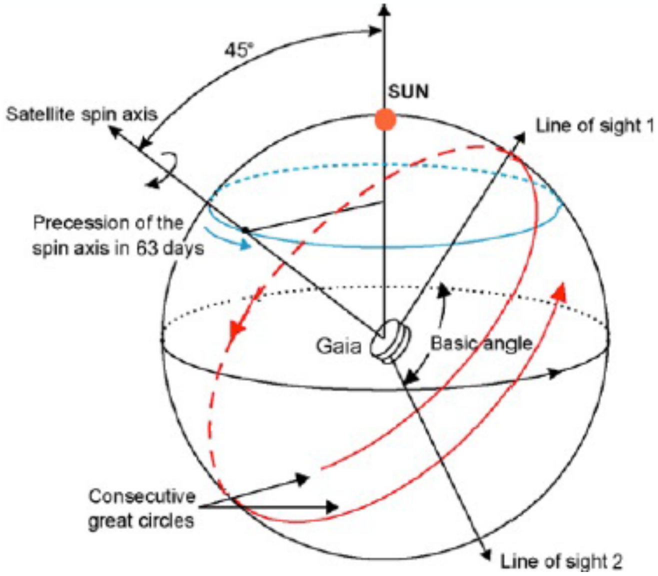
M=number of observations

- Gradient-type iterative method LSQR (based on Lanczos bidiagonalization algorithm)
  - Works well on sparse matrices
  - More flexible to small model changes
  - Computationally less expensive (15M+7N)*k [k=number of iterations]
  - Vectorial optimization of the initial code was performed on CRAY X-MP/12 @CINECA

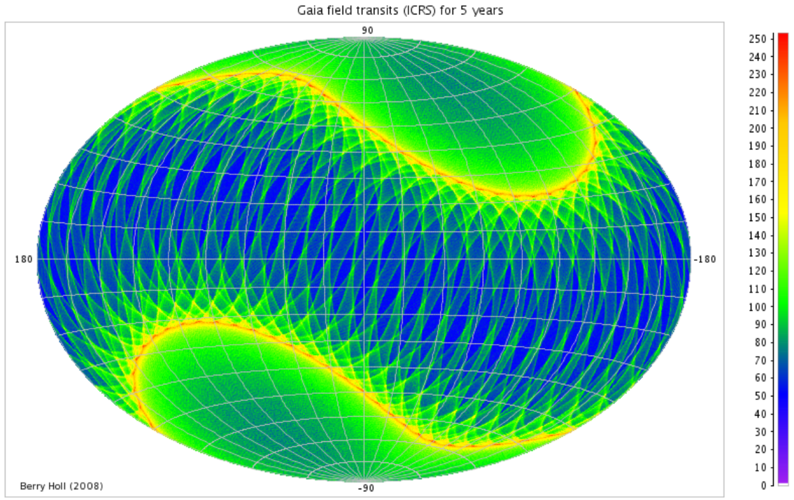
ESA Gaia Mission



- Launched on Dec 2013 in a Lissajous-type orbit around lagrangian point L2 (1.5M Km from Earth)
- Conducting an optical all-sky survey to magnitude 20 (~1% of Galaxy stellar content)
- Observing principles based on predecessor HIPPARCOS but 3 order of magnitude more accurate
  - Spacecraft slowly rotating at angular rate of 1° per minute
  - Spin axis slowly precesses around Sun direction with 63-day period
  - Full sky coverage obtained every 3 months
- Focal plane assembly largest ever developed for space applications
  - 106 CCDs 4500x1966 px^2 each (total of ~1000M), 1x0.4m^2 physical dimension
  - 10x30 micron^2 pixel size
  - CCD operate in Time Delay Integration (TDI) mode
    - Photoelectrons are clocked across CCD together with moving star image



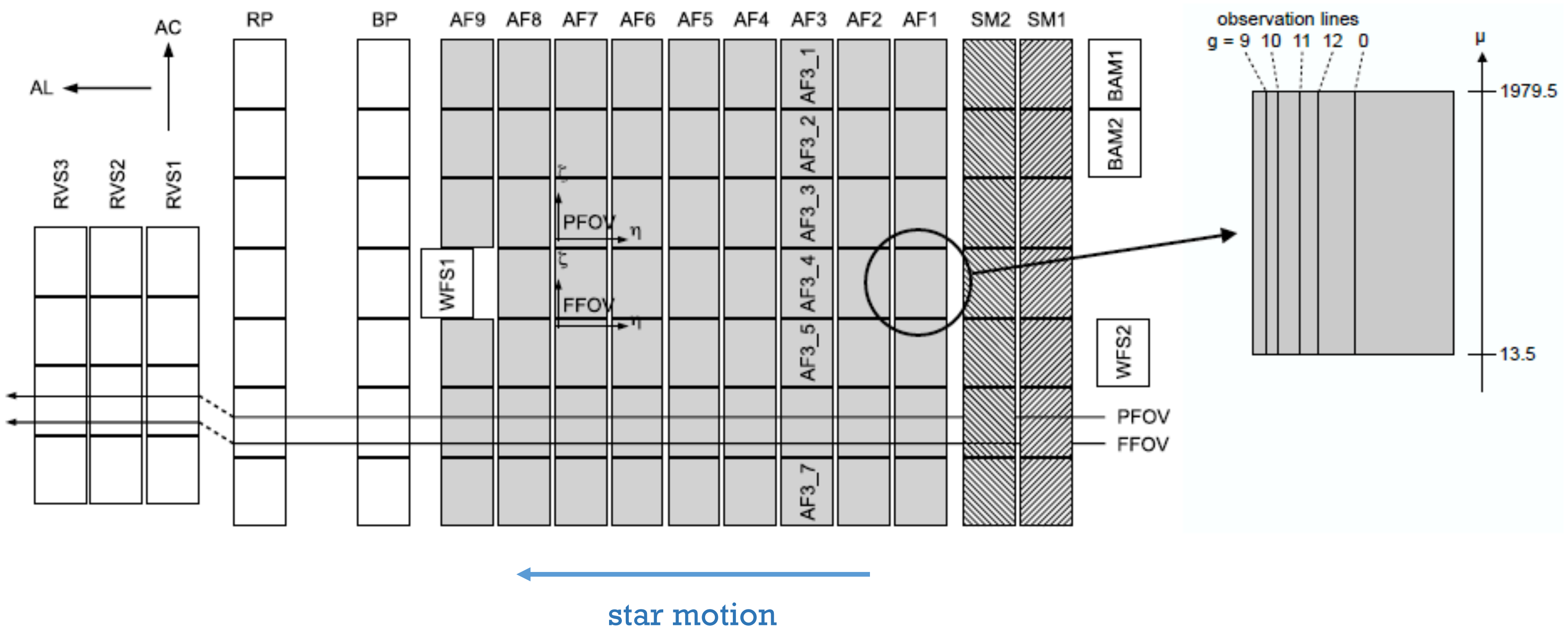
Gaia scanning law

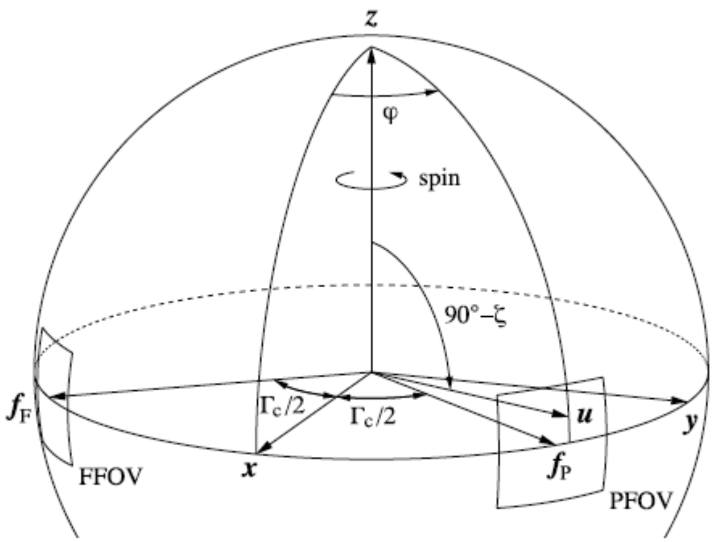


in ecliptic cordinates

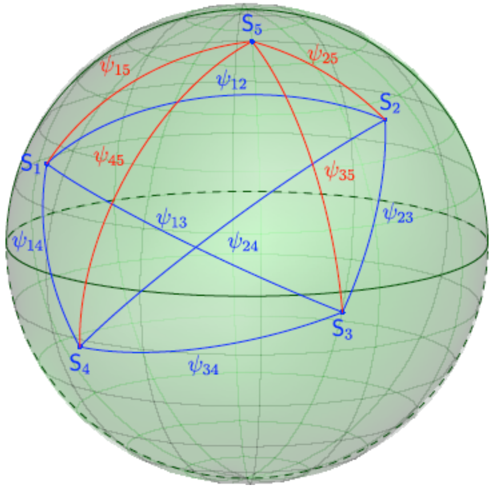
Gaia Astrometric Observation

- The astrometric observation is the precisely estimated instant when the star image center crosses the nominal CCD ‘observation line’
- The timing observations provide accurate (~0.1 to 1 milliarcsecond) information about the instantaneous relative along-scan position of the observed objects



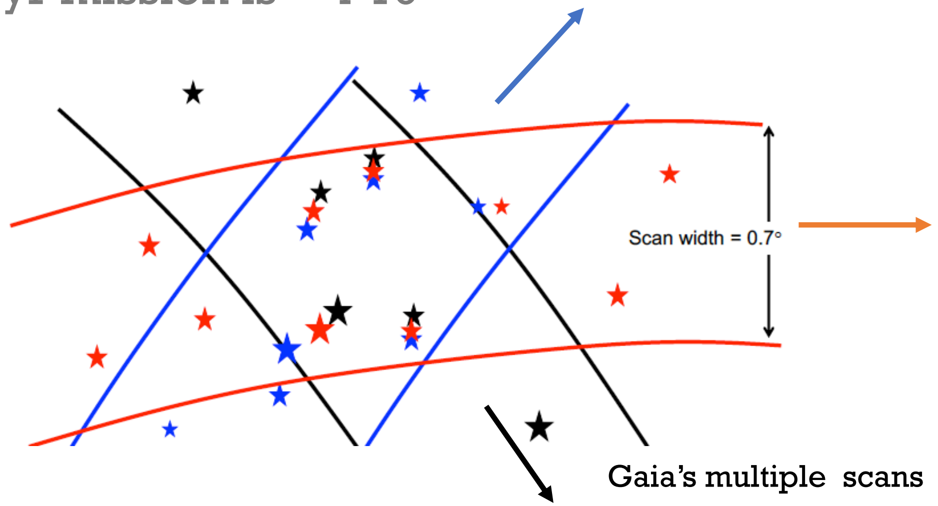


Gaia Sphere Solution (1)



- Stars' along-scan measurements create a geodetic-like network on the celestial sphere
- Problem dimensions:
  - ~ 10^8 primary stars, each observed ~80 times in 5 yr , i.e. ~720 CCD transits, a total of 7.2 10^{10} observations.
  - Number of stellar parameters 5 10^8
  - Number of attitude parameters to be estimated for a 5-yr mission is ~ 4 10^7
  - Number of instrumental calibration parameters ~10^6
- General form of minization problem:

$$\min_{(s,n)} \left\| \boldsymbol{f}^{\text{obs}} - \boldsymbol{f}(\mathbf{s},\mathbf{n}) \right\|$$



Gaia's multiple scans

s = vector of unknown stellar parameters describing their barycentric (Sun centered) motion

n = vector of «nuisance parameters» describing the instrument

$\mathbf{f^{obs}}$ = vector of the observations

f(s,n) = observation model, i.e., expected detector coordinates calculated as function of astrometric and nuisance parameters

Gaia Sphere Solution (2)

- The minimization problem corresponds to the least-squares solution of the overdetermined system of equations

$$f_l^{obs} = f_l(\boldsymbol{s}_i, \boldsymbol{n_j}) \quad l = 1, \ldots, \text{n. of observations}$$

  - index $\boldsymbol{i}$ indicates the stellar source, index $\boldsymbol{j}$ indicates the set of nuisance parameters

- The $\boldsymbol{f}$ function is highly non-linear, but initial errors in **s** and **n** are small
  - second-order terms of linearized equation typically less than 10^{-12} rad (~ 0.2 micro-arcseconds) → negligible compared with single-observation noise
- The observation equation is conveniently linearized around suitable initial values:

$$f_l^{obs} - f_l^{calc} = \frac{\partial f_l}{\partial \boldsymbol{s}_i} \boldsymbol{x}_{si} + \frac{\partial f_l}{\partial \boldsymbol{n}_j} \boldsymbol{x}_{nj}$$

- The weighed least-squares system is formed multiplying each equation by the square root of its statistical weight (the inverse of its standard deviation)

Gaia sphere solution : matrix structure

- Considering for simplicity only stellar **s** and attitude **a** unknowns, the linearized system of equations in matrix notation reads $O\mathbf{x} = \mathbf{b}$, $\mathbf{x}=\mathbf{x}(\mathbf{s},\mathbf{a})$
  - sorting observations by the **n** stellar sources one gets a block angular matrix

$$\left[\begin{array}{ccc|c} S_1 & 0 & 0 & A_1 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & S_n & A_n \end{array}\right] \begin{pmatrix} x_{s1} \\ \vdots \\ x_{sn} \\ x_a \end{pmatrix} = \begin{pmatrix} b_{s1} \\ \vdots \\ b_{sn} \end{pmatrix}$$

  with block dimensions

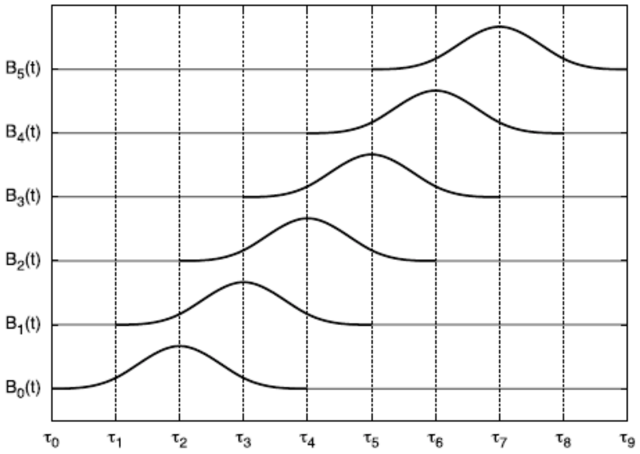
$\boldsymbol{S}_i(o_{si}\times 5)$, $\boldsymbol{A}_i(o_{si}\times m)$ and $\boldsymbol{x}_{si}(5\times 1)$, $\boldsymbol{x}_a(m\times 1)$, $\boldsymbol{b}_{si}(o_{si}\times 1)$

  where $\boldsymbol{o}_{si}$ is the number of observations of the i-th source and $\boldsymbol{m}$ is the number of attitude parameters

- $\boldsymbol{S}_i$ are full matrices, while $\boldsymbol{A}_i$ are very sparse with $[A_i]_{\alpha\beta} \neq 0$ only if the αth observation of source *i* is linked to the βth attitude parameter.
  - Each source observed relative to large number of other sources simultaneously in the two FOVs, linked togheter by the attitude (and calibration) model → strong connectivity of matrix structure → does not allow sequential processing

Gaia sphere solution : computational complexity

- Sparseness structure of condition matrix *O* directly related to the choice of spline function representing the Gaia satellite attitude:
  - piecewise polynomial function written as linear combination of B-splines of order M=4 (cubic) defined on a sequence of M+1 time knots
- At any observing time t_i there are only 4 non-zero cubic B-splines, and the associated spline coefficients to be estimated are $a_{i-M+1}, a_{i-M+2}, \dots, a_i$
  - the sub-vectors $\boldsymbol{a}j$ consist of 3M scalar values, i.e., M spline coefficients for each of the three orientation angles of the satellite axes.
- The observation equation for different sky sources (sub-vectors s_j) may refer to the same attitude sub-vectors a_j



Cubic B-spline sequence

<u>Fill factors (fraction of non-zero elements)</u>

- Sub-matrix *A* fill factor 3M/m=12/m (m=n. of attitude params)
- Full matrix *O* fill factor (5+3M)/(5n+m),
  with $n=10^8$, $m=4\ 10^7$, M=4 → fill factor of *O* ~ $2\ 10^{-8}$

Gaia sphere solution : numerical approach

- The least-squares normal equations are $O^T O\boldsymbol{x} = O^T\boldsymbol{b}$ ($O^T O \equiv N$) with solution $\boldsymbol{x} = (O^T O)^{-1} O^T\boldsymbol{b}$ and structure of the normal matrix *N* of dimension *(5n+m)*:

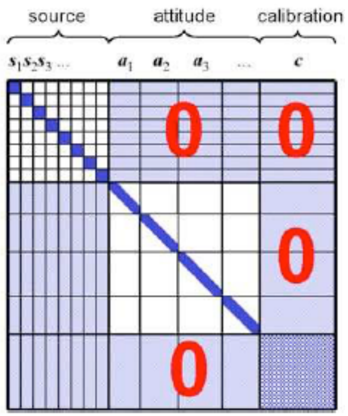
$$\begin{bmatrix} S_1^T S_1 & 0 & \dots & 0 & S_1^T A_1 \\ 0 & S_2^T S_2 & \dots & 0 & S_2^T A_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & S_n^T S_n & S_n^T A_n \\ A_1^T S_1 & A_2^T S_2 & \dots & A_n^T S_n & \sum A_i^T A_i \end{bmatrix} \begin{pmatrix} \mathbf{x}_{s1} \\ \mathbf{x}_{s2} \\ \vdots \\ \mathbf{x}_{sn} \\ \mathbf{x}_a \end{pmatrix} = \begin{pmatrix} S_1^T \mathbf{b}_{s1} \\ S_2^T \mathbf{b}_{s2} \\ \vdots \\ S_n^T \mathbf{b}_{sn} \\ \sum A_i^T \mathbf{b}_{si} \end{pmatrix}$$

  Matrix *N* has a **doubly bordered block diagonal** form, with a **block size** of **5** and **border width** *m*

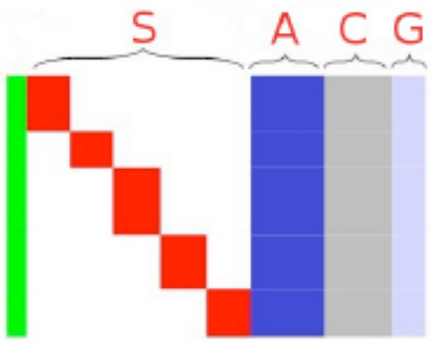
  - the dimension of the sub-matrices are $S_i^T S_i$(5x5), $S_i^T A_i$(5xm), $\sum_i A_i^T A_i$ (mxm)
  - the fill factor of *N* is ~ (310)/n→ 3 10^{-6}.
- Standard successive elimination of the unknowns along the block diagonal leaves a *reduced normal matrix* of much smaller dimension but much denser
  - Bombrun et al. (2010) investigated the Cholesky factorization of the reduced normal matrix concluding that a direct solution of the reduced normal equations for a 5-yr (7300 spin periods) mission would require about 10^{21} flops ($m^3/6$ operations for Cholesky decomposition of full *mxm* matrix)
  - the dimension of the upper triangular matrix would require ~ 2M GigaBytes memory
- Direct approach practically unfeasible

Gaia sphere solution : numerical approach

- Gaia's pipeline (AGIS): rigorous solution via a block-iterative technique solving separately each block of unknowns and disregarding the cross-terms connectivity
  - Covariances of stellar parameters are estimated at the last iteration, neglecting statistical correlations introduced by the attitude and calibration models (Holl and Lindegren 2012)
- Alternative solution method (GSR) implemented by the Astrometric Verification Unit (AVU) (Vecchiato et al. 2018) makes use of the LSQR method
  - Iterative algorihm (Paige & Saunders 1982) similar to the method of conjugate gradients
  - works on the condition matrix *O* and efficiently treats its sparseness
  - It provides an estimation of the unknowns' standard deviations
  - The original LSQR code has been modified to estimate the covariances of any selected group of unknowns

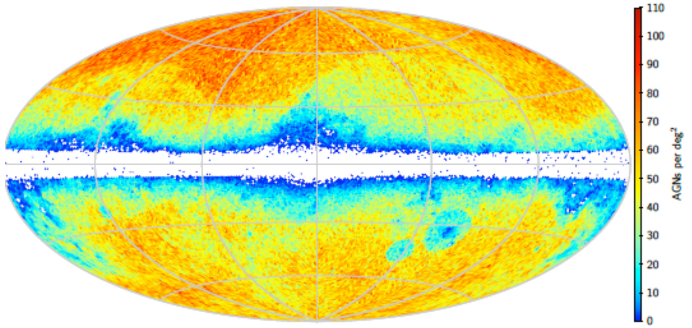


Structure of AGIS (normal) matrix

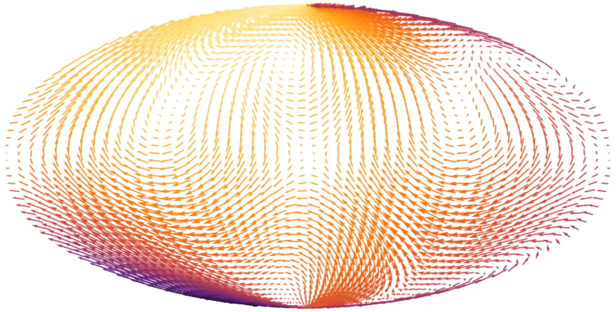


Structure of GSR (condition) matrix

  - Given the data volume and computational complexity, HPC tecniques are being exploited to optimize the code (Cesare et al. 2022)



Search for Cosmological Signatures



- Proper motions of extragalactic objects such as Quasars(QSO) can reveal a variety of cosmological and observer-induced phenomena over a range of angular scales (Darling at al. 2019)
  - Secular aberration, secular extragalacitc parallax, Gravitational Waves, anysotropic expansion
- **Vector Spherical Harmonic (VSH) Analysis** is a powerful tool for their investigation
  - The proper motion vector field $\vec{\mu}$ of QSOs can be decomposed into a set of orthogonal basis functions on the sphere (VSH) as

$$\vec{\mu} = \sum_{l,m} \left(t_{l,m} \boldsymbol{T}_{l,m} + s_{l,m} \boldsymbol{S}_{l,m} \right)$$

where $\mathbf{T}_{lm}$ and $\mathbf{S}_{lm}$ are respecitvely the toroidal and spheroidal base functions of degree l and order m

- The residual proper motion field, defined on the sphere surface, orthogonally to the radial direction, is $\mathbf{V}(\alpha, \delta)$. Such field can be expanded in a unique linear combination of VSH functions as

$$\mathbf{V}(\alpha, \delta) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \left(t_{lm} \boldsymbol{T}_{lm} + s_{lm} \boldsymbol{S}_{lm} \right)$$

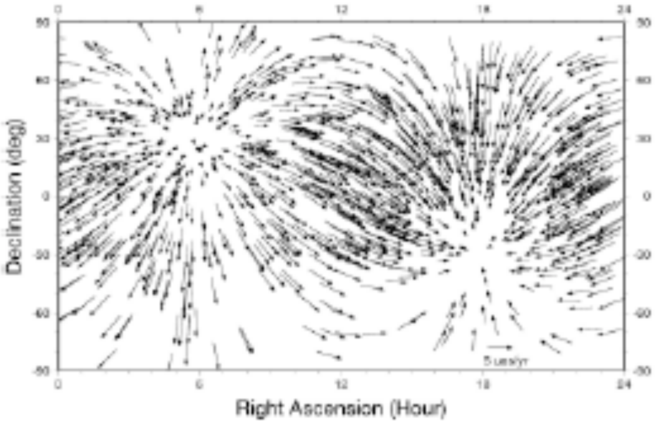
where practically the expansion is truncated to a certain degree *l*.

Search for Cosmological Signatures

Some Numerical Aspects

- The coefficients t_{lm} and s_{lm} are estimated in a least-squares adjustment, where the degree l is reflecting the angular resolution of the field systematics that are probed ($\theta \sim \pi/l$)
- Given N sources (~ 1.6M QSOs found in Gaia DR3) and a truncation to degree L, the condition equation system has dimensions $2N \times 2L(L+2)$
  - the data storage of the full design matrix could be a problem, so it is convenient to build up the normal matrix on the fly; for a large dataset, ($N >> l^2$), this step is the most demanding in terms of computing time.

- Signals due to residual rotation and acceleration of the reference frame materialized by QSOs are fully contained in the first degree harmonics
- Stochastic gravitational waves can also mimick a proper motion field whose sky average $\langle \mu^2 \rangle$ can be directly related to the energy density of the cosmological GW background and is mostly contained in the degree 2 of VSH expansion (Gwinn 1997)
  - The amplitude of such signatures is expected to be < 1 micro-arcsecond → difficult for Gaia, certainly within reach of the next-generation astrometric missions



Dipole pattern of QSOs proper motion induced by Galactic aberration (~5 micro-arcseconds)