The Problem of the Global Astrometric Sphere Reconstruction in Astrometry

Problems and approaches

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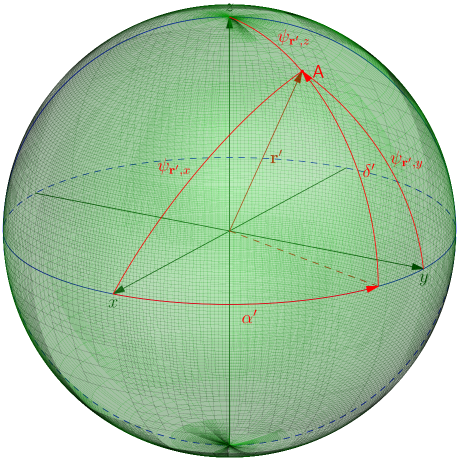
What is astrometry about

- Measure the "apparent" (i.e. local) position (α', δ') or ψ_{r', e_a} of a star.
- Get its "true" (i.e. barycentric) position (α, δ) from them. This requires the application of some "corrections".

Mathematical modeling

Write the equations that connect the observables (i.e. $\cos \psi_{r', e_a}$) with the unknowns (i.e. α and δ)



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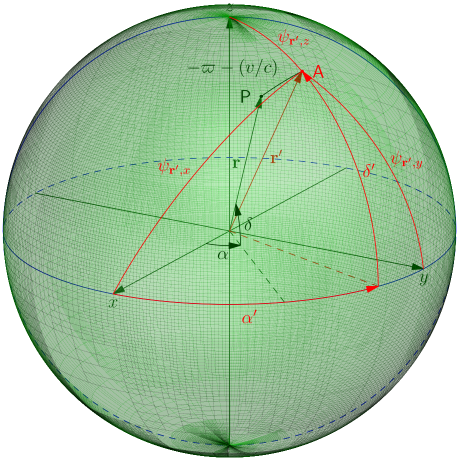

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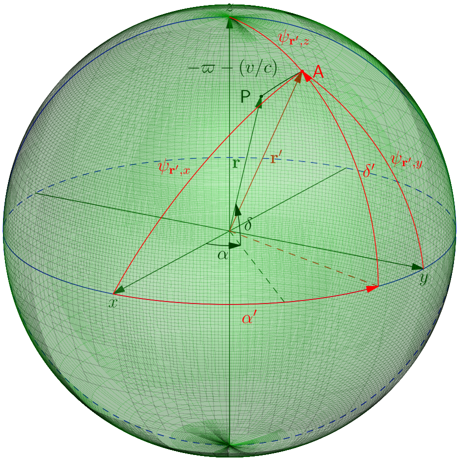
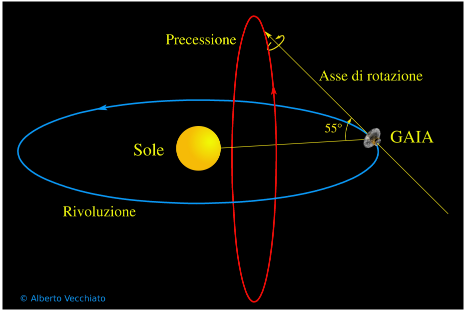
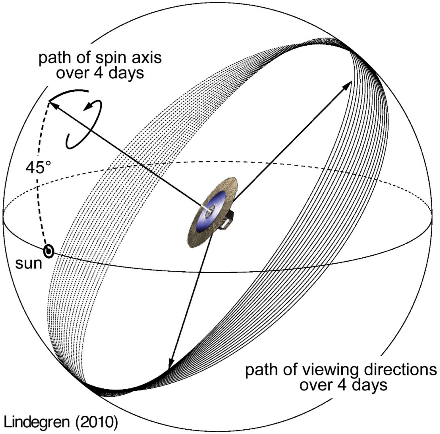


Absolute Global astrometry with Gaia

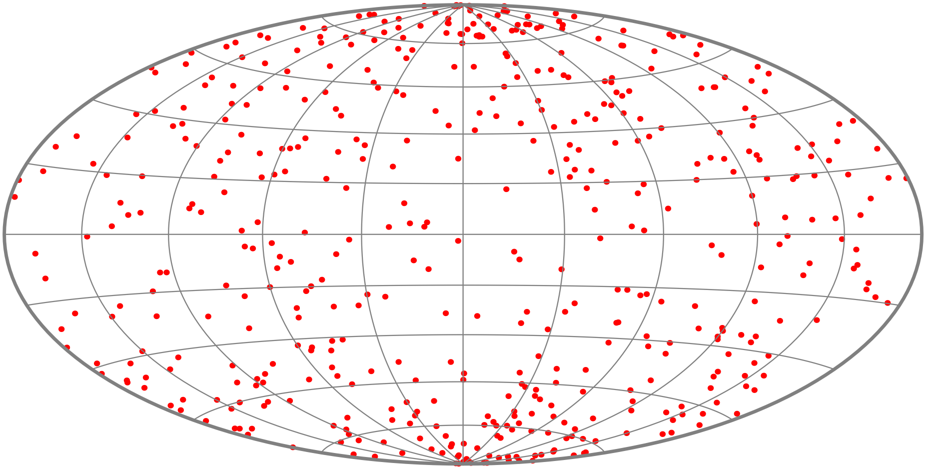
Building on the legacy of the Hipparcos satellite, Gaia implements an approach to global and absolute astrometry via a two-way telescope doing measurements in scanning mode





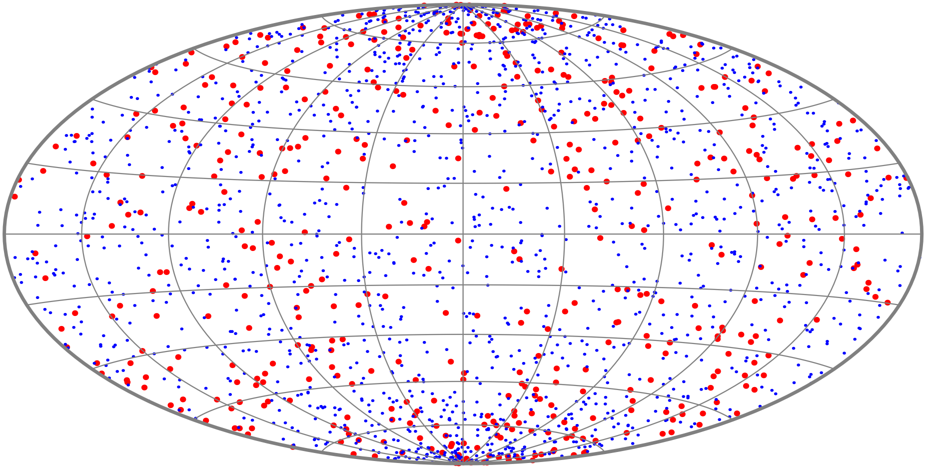
Primary and secondary sources in Gaia

The Global Astrometric Sphere is first reconstructed with respect to a subset ($\sim 10^8$ out of $\sim 10^9$) of well-behaved stars called primaries.

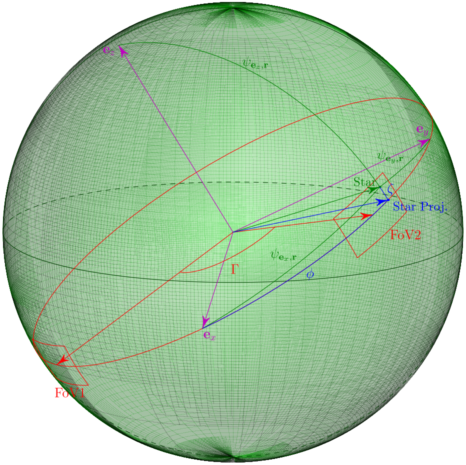


Primary and secondary sources in Gaia

The reference frame materialized by the primaries is used by other pipeline processes to include the other stars into the Gaia sphere.



Mathematical modeling: the Euclidean abscissa



- Gaia basic observable: abscissa ϕ between the x axis and one viewing direction

$$\cos\psi_{(\hat{a},\mathrm{r})} = \frac{\mathrm{e}_{\hat{a}}\cdot\mathrm{r}}{|\mathrm{r}|} \qquad (1)$$

$$\cos\phi = \frac{\cos\psi_{(\hat{x},\mathrm{r})}}{\sqrt{1-\cos^2\psi_{(\hat{z},\mathrm{r})}}} \qquad (2)$$

- Depends on: Astrometric (S), attitude (A), calibration (C), and global (G) parameters

$$\cos\phi = \frac{\cos\psi_{(\hat{x},\mathrm{r})}}{\sqrt{1-\cos^2\psi_{(\hat{z},\mathrm{r})}}} = F\left(\boldsymbol{x}^{\mathrm{S}},\boldsymbol{x}^{\mathrm{A}},\boldsymbol{x}^{\mathrm{C}},\boldsymbol{x}^{\mathrm{G}}\right)$$

The Linearized system of equations

- A first-order Taylor expansion around a convenient set x_0 of starting values (catalog) of the unknown parameters $x \equiv \{x^S, x^A, x^C, x^G\}$ linearizes the observation equations and the equation system

$$-\sin\phi_{calc}\,\delta\phi = \sum_{Source} \left.\frac{\partial F(x)}{\partial x^S}\right|_{x_0} \delta x^S + \sum_{Attitude} \left.\frac{\partial F(x)}{\partial x^A}\right|_{x_0} \delta x^A + \sum_{Cal} \left.\frac{\partial F(x)}{\partial x^C}\right|_{x_0} \delta x^C + \sum_{Global} \left.\frac{\partial F(x)}{\partial x^G}\right|_{x_0} \delta x^G$$

$$\delta\phi = \phi_{obs} - \phi_{calc}$$
$$\delta x = x_{true} - x_0$$
$$\phi_{calc} = F(x_0)$$

- The new unknowns are the corrections to the catalog values. Their estimation $\bar{\delta x}$ gives

$$x_{true} \simeq \bar{x} = x_0 + \bar{\delta x}$$

- The resulting $m \times n$ system of equations is:
  - sparse $\Rightarrow$ #of $(a_{ij} \neq 0) \ll m \times n$
  - overdetermined $\Rightarrow$ $n \ll m$

Solving the Equation System

The NON-feasibility of Direct methods

- Linear System of Equation: $b = Ax$, sparse, overdetermined

$$x = \left(A^T A\right)^{-1} A^T b$$

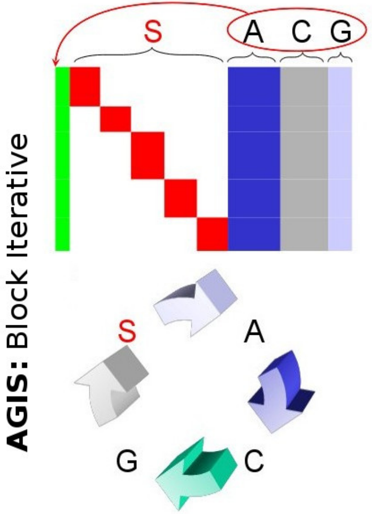
- Direct methods: needed operations$\propto N_{\text{unk}}^3 \sim 2 \cdot 10^{26}$

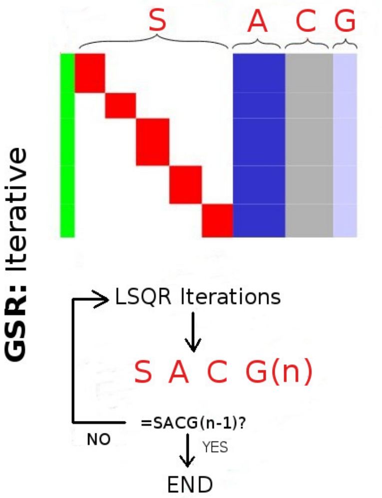
Most powerful supercomputer as of November 2023: Frontiers, DOE/SC/Oak Ridge National Laboratory, USA,1.6 EFlop/s

⚠ Time needed: 2 years

Solving the Equation System

Parallelized and iterative approaches are mandatory for solving the equation system.



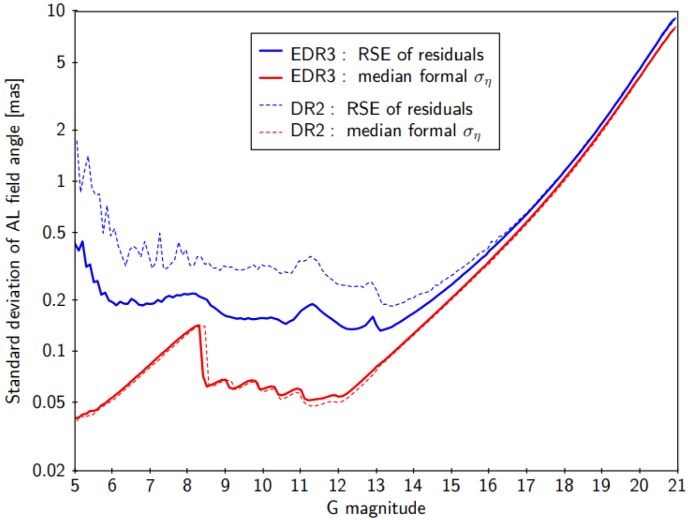


Current status of the Gaia catalog

- The DPAC (i.e. the consortium in charge of the reduction of Gaia data) has already released 3 versions of the astrometric catalog
- The latest release (DR3) features:
  - full astrometric solution (5- or 6-parameters) for 1.46 billion sources in the magnitude range $3 \lesssim G \lesssim 21$, and 2-parameters solution for another 3.34 million sources
  - G magnitude for about 1.8 billion sources
  - G_{BP} and G_{RP} for more than 1.5 billion sources
  - astrophysical parameters (T_{eff}, $\log g$, $[M/H]$, etc.) from BP/RP spectra for 470 million sources
  - astrophysical parameters from RVS spectra for 5.5 objects
  - ...

Most important known issues of the astrometric catalog

- Parallax bias of QSOs (zero-point parallax error) of $-17\ \mu$as [3]
- Magnitude-, color-, and position-dependent parallax biases [2]
- Measurements' residuals not following the expected magnitude dependence



Possibile origins of the issues

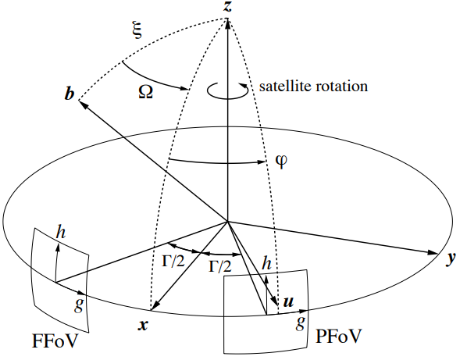
- The most probable causes of these issues are a suboptimal calibration and the strong correlations that exist among different kind of unknowns.
- Most significant correlations:
  1. Correlation between Attitude and Basic Angle variations.
  2. Correlation between parallaxes and Basic Angle variations.
  3. ~~Correlation between parallaxes and light deflection (PPN parameter γ).~~
  4. Correlation between Basic Angle variations and other calibration parameters.
- It has to be stressed that Gaia is a *self-calibrating instrument*, namely,

The calibration parameters are unknowns of the global astrometric sphere reduction!

Correlation between Attitude and Basic Angle variations (I)

- The attitude in Gaia is geometrically defined by the directions of the Fields of View (FoV)
- Therefore, e.g., a rotation around the z axis of one of the FoVs, is equivalent to a rotation around the same axis of the x and y attitude axes.
- This issue is benign until it does not influence the astrometric parameters but...
- Large number of attitude unknowns $\sim 10^6 - 10^7$, avoidable is arc measurements are feasible instead

$$\cos\psi = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1|\,|\mathbf{r}_2|}.$$



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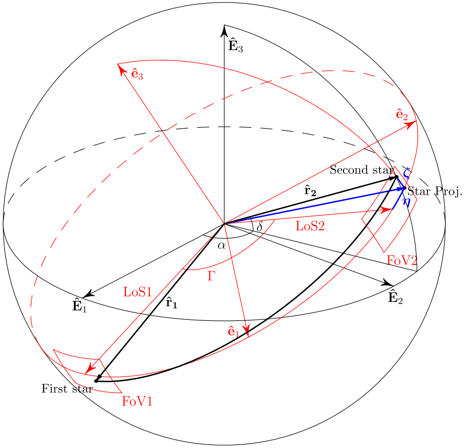
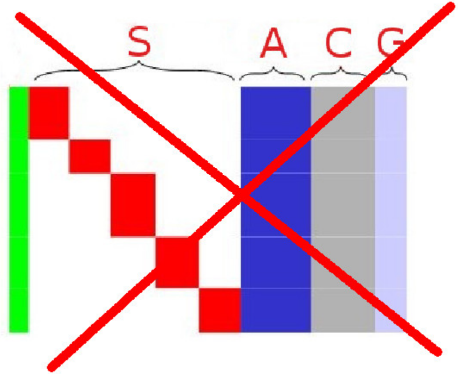


Feasibility of arc-based approach in Gaia and Gaia-like missions

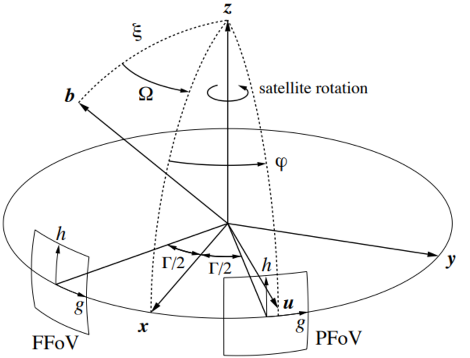
- An arc observable can in principle be modelled in Gaia and Gaia-like missions.
- Constraints the accuracy of the along- and across-scan rates σ_{ω_η} and σ_{ω_ζ} respectively.
- Block-diagonal rearrangement of the astrometric part of the design matrix disrupted. Needs a completely different parallelization algorithm.



Correlation between parallaxes and Basic Angle variations (I)

It has been shown [1] that a change in the parallax $\delta\varpi$ can be fitted by a specific combination of attitude and Basic Angle variations (R is the distance between Gaia and the barycenter of the Solar System):

$$
\begin{aligned}
\delta x &= 0 \\
\delta y &= \cos\xi \, \cos^{-1}(\Gamma_n/2) \, R \, \delta\varpi \\
\delta z &= \sin\Omega \, \sin\xi \, \cos(\Gamma_n/2) \, R \, \delta\varpi \\
\delta\Gamma &= 2\cos\Omega \, \sin\xi \, \sin(\Gamma_n/2) \, R \, \delta\varpi
\end{aligned}
$$



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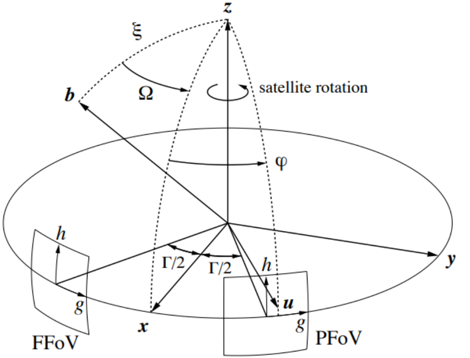
⚠ **Correlation with Parallexes zero-point.**



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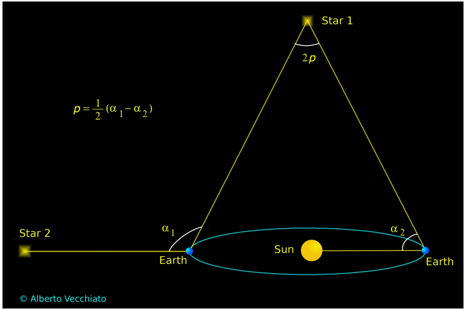


Correlation between parallaxes and Basic Angle variations (II)

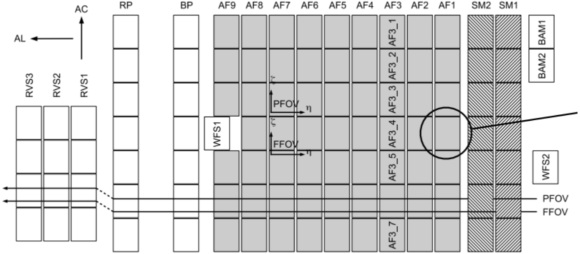
- The expression of the correlation is completely degenerate only with the $a_1^{(\Gamma)}$ coefficient of the harmonic expansion of a generic perturbation

$$\delta\Gamma = \sum_{k\leq 0} a_k^{(\Gamma)} \cos k\Omega + b_k^{(\Gamma)} \sin k\Omega.$$

- Other coefficients can be included as unknowns of the calibration model.
- The Basic Angle Monitoring (BAM) instrument provides an independent calibration of the BA short period variations at the μas level.

Correlation between Basic Angle variations and calibration (hints)

- The astrometric focal plane of Gaia is made of 62 CCDs.
- Each CCD can be displaced from its nominal position, inducing a systematic error in the estimation of the AL and AC measurements that depends on the FoV $(F_{(f)}(t), F_{(p)}(t))$.



- A non-zero average of the displacements is thus completely degenerate with:
  - The AL attitude $\left(\left(F_{(f)}(t)+F_{(p)}(t)\right)/2\right.$.
  - The Basic Angle variation $(F_{(f)}(t)-F_{(p)}(t))$.
- Other significant systematic errors can derive from unmodelled dependencies of the measurements from the color and magnitude of the source, saturation, subpixel phase.

⚠ More than 10^6 calibration parameters in the EDR3 solution!

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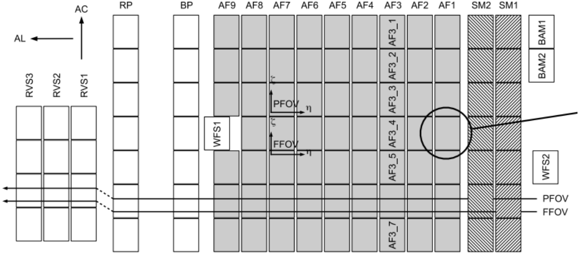

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Summary and conclusions (I)

- The reconstruction of the Global Astrometric Sphere translates into the solution of a big system of linear(-ized) equations.
- Due to its size, the problem can be approached only with iterative algorithms.
  - Block-iterative methods are easier to implement but can easily underestimate the covariances between unknowns of different blocks.
  - Fully-iterative methods require a more complex parallelization, but are more reliable on the variance-covariance estimation.
- A Gaia-like scanning satellite inevitably introduces some strong correlations among the unknowns of different blocks.
  - Correlations between Attitude and BA are generally uneventful, unless they couple with the astrometric parameters $\rightarrow$ calls for μas-accurate independent reduction of BA variations.
  - Correlations between parallaxes and light deflection does not prevent the estimation of parallaxes, and cannot introduce a significant zero-point parallax error. However, it can prevent a scientifically useful estimation of the PPN parameter γ.

Summary and conclusions (II)

- The most delicate issues are likely to originate from uncalibrated instrument effects.
- Connection with the covariance estimation/solution methods of the equation system?
- The Gaia experience taught us that the self-calibration philosophy followed until now, in the future might benefit from better independent instrument calibrations:
  - dedicated calibration payload?
  - different measurement approach?
- Alternative approaches to the sphere reconstruction (arc-based?) that try to get rid of some "nuisance parameters" can also help to alleviate these problems.

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